INTEGRATING AND DECIPHERING SIGNAL BY FOURIER TRANSFORM

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ABSTRACT

The Fourier transform can be used to solve the periodic wave equation and non-periodic, this has been done by Frediric J. Harris. To provide an understanding and confidence in scientists who are beginners learn Fourier transformation, it will be required a proof that the integrated signal wave when deciphered using Fourier transformation will have parameters whose value is equal to the basic wave signal parameters before the integrated. This research uses an application program MS. Excel to integrate some basic signals, and use Matlab to decompose the results of the combination of these signals, then the results will be analyzed.

To answer the problems and aims of research are required steps: 1) Combine multiple sinus equation by giving value to the parameters sine equation: $Y = A \sin(\omega t + \theta)$, and then the Y value is calculated along the value t squen, repeat previous process as much as desired with a value of A, ω, and θ is different, and each the value of Y will be summed, so it will be formed a blend of discrete wave. 2) Decipher the integrated wave signal to obtain parameters of the fundamental frequency (fo), k, and the amplitude (A). The results of each parameter will be compared to the parameters of the basic signal.

The examination results showed that the decomposition wave signal integrated using Fourier transform obtained the values of the parameters of frequency (f), the coefficient of frequency (k), and amplitude (A), the value is almost equal to the initial state before, with an error rate of only 0.0143 or 1.43 %.

Keywords: Fourier transform, integrated wave signal, frequency, deciphering, integrating

INTRODUCTION

Fourier is one part of the mathematical sciences were invented by a French scientist named Jean-Baptiste Joseph Fourier (1768-1830). First Fourier series is found in the form known as a Fourier series with sinusoidal functions (sine and cosine). In general, this series used as tools to solve differential equations, both ordinary differential equations and partial differential equations. In the telecommunications sector, the role of Fourier very important one of which is used to decipher the signals from the result of the combination wave, to wave with periodic time can be solved with the aid of a Fourier series, but for a wave of non-periodic Fourier series was not able to finish, according to its inventor Joseph Fourier can solved by Fourier transformation. The Fourier transform can be used to solve the wave equation periodic and non-periodic, this has been done by Fredíric J. Harris. The frequency of the first, second, third, and so on of a signal result of the combination wave sine / cosine periodically obtained from Fourier transformation, as well as Fourier transformation is exactly the FFT (Fast Fourier Transform) can be used to find the equivalent noise bandwidth of a signal (Harris, 1998)

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The Fourier transform is widely used to solve problems related to signal processing, including the use of spectral methods for computing on-line component symmetrical hamonik based on computational DFT (Discrete Fourier Transform), indicating that this method can be used to detect electrical faults on the load or on power lines with normal operation (H.Henao, 2003). Application FFT algorithm in the processor TMS320C542 to calculate the signal frequency spectrum sine, triangle, and a square with a specific sampling frequency, the result of this study is, the frequency response of the results of the implementation was consistent with the theory, namely that the sine signal there is only the fundamental frequency. While a triangular signal and square signal are the fundamental frequencies and harmonic frequencies that appear at odd multiples of the fundamental frequency (Damayanti, 2010).

Almost all Fourier teori and aplication is so difficult for understanding it, so in this research we want to explain about Fourier which will be implemented for integrating and deciphering signal. We will use simple method for integrating and deciphering signal, using MS. Excel and Mathlap.

INTEGRATING SIGNAL

A periodic signal should have formulation :

$$y(t) = y(t + nT), dengan\, n = 0, \pm 1, \pm 2, \dots \qquad (1)$$

For all of t (time), and T is time period as the formulation start to repeat or oscillation. In the sine function is given by y(t) = A sin (ωt + θ), every function that periodically turns can be expressed by a superposition of sine and cosine functions. It is known that the trigonometric functions sin(ωt) and cos(ωt) are periodic with period T = 1 / f = 2π / ω, where f is the frequency in cycles per second (Hz) and ω is the angular frequency in radians / sec. Figure 1 shows a periodic function, the fundamental period $T_0 = 2\Pi / \omega_0$. where ω_0 is the fundamental frequency, below is example of periodic signal.

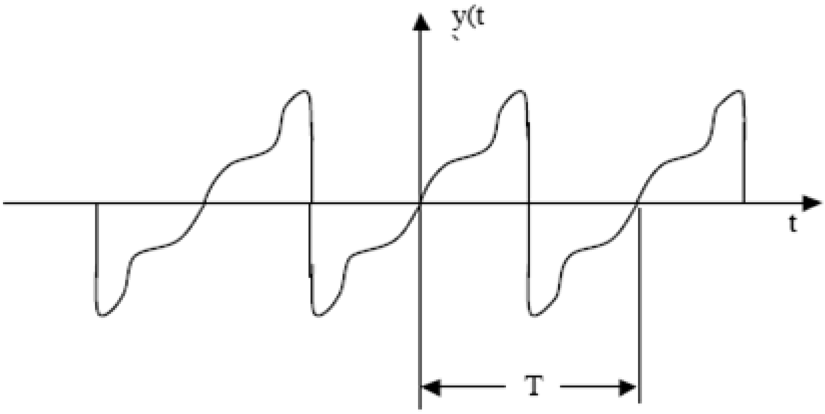

Figure 1. Periodic Signal

According to (Hayes, 1995) that Fourier trigonometry series has formulation, following:

$$y(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos k\omega_0 t + b_k \sin k\omega_0 t) \qquad (2)$$

By assuming that sine wave is natural signal which is produced by source signal such as light, heat, sound, and so on, in the general can be written by formulation : y(t) = A sin (2πft + θ). Every paramters amplitude (A), linear frequency (f), and time (t) from 0 until 360, so we will obtain a sine graphs which shows relation between y(t) versus t. If there are two or more of sine graphs and then every parameters summed so that will produce a integrated sine graph. The square signal is one of simple integrated wave, equation (3) is formulation of integrating signal (Yarlagadda, 2010).

$$X\ [t] = \sum_{k=1}^{K} \frac{A}{\pi k} * \sin(2\pi k f.nt), dengan\ k = 1,2,3,4,5 \ldots. \qquad (3)$$

by,

x[t]	: function time domain
A	: Amplitude
f	: frequency
k	: index of frequency multiple
n	: n sampling
t	: time sampling

Equation (3), almost all theory give value of k is odd, but in this research try to give odd, even, and random value. The Figure 2 shows flowcahrt of integrating signal,

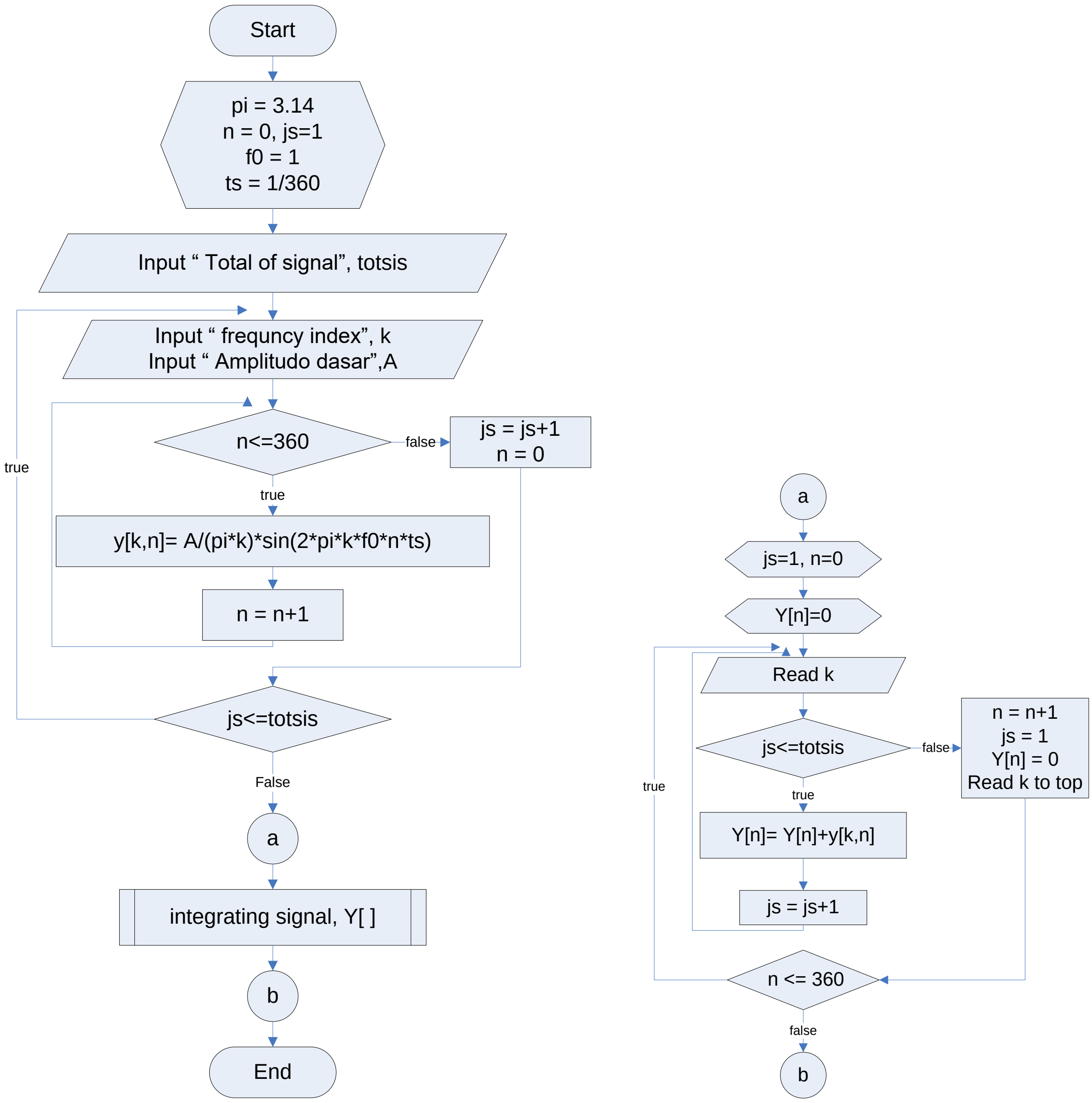

Figure 2. Integrating Signal Diagram

On figure 2, f_0 is foundation frequency, ts is sampling time, totsis is signal total. On flowchart figure 2, the integrating process uses summing operation, for replacing of process like multiplication, division, convolution, and etcetera, just replace the operator.

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DATA AND PROCESS OF INTEGRATING SIGNAL

According to Yarlagadda when k is odd (1, 3, 5, …) so the integrated wave will form square. There are three variate data in this research, namely odd, even, and random, the integrating process uses summing, here is example of calculation:

- Parameter forming sine signal, fo = 1Hz, A = 220, k = 1, n = 1..360, ts = 1/360 sec; so the equation y[1,n]

  $$y[1,n] = \frac{220}{3.14*1} * \sin\left(2*3.14*1*1*n*\frac{1}{360}\right), by\ \ n = 1..360$$

- Parameter forming sine signal, fo = 1Hz, A = 220, k = 3, n = 1..360, ts = 1/360 sec; so the equation y[3,n]

  $$y[3,n] = \frac{220}{3.14*3} * \sin(2*3.14*3*1*n*1/360), by\ \ n = 1..360$$

- Parameter forming sine signal, fo = 1Hz, A = 220, k = 5, n = 1..360, ts = 1/360 sec; so the equation y[5,n]

  $$y[5,n] = \frac{220}{3.14*5} * \sin(2*3.14*5*1*n*1/360), by\ \ n = 1..360$$

the integrating signal Y[n] is summing from every y[k,n] value, here is the example data :

Table 1. Example Odd Data Index

n	y[1,n] k=1	y[3,n] k=3	y[5,n] k=5	**Y[n]**= y[1,n] + y[3,n]+ y[5,n]
0	0	0	0	0
1	1.22216	1.221664	1.220672	3.6645
2	2.443948	2.439979	2.432053	7.316
3	3.664991	3.651607	3.624925	10.942
4	4.884919	4.853225	4.790209	14.528
5	6.103358	6.041541	5.919036	18.064
6	7.319937	7.213298	7.002816	21.537
7	8.534288	8.365284	8.0333	24.934
8	9.746038	9.49434	9.002645	28.244
9	10.95482	10.59737	9.903475	31.457
10	12.16026	11.67136	10.72893	34.563
	.			
351	-10.9538	-10.5965	-9.90275	-32.03
352	-9.74502	-9.4934	-9.00186	-28.84
353	-8.53327	-8.36432	-8.03246	-25.54
354	-7.31891	-7.21232	-7.00192	-22.16
355	-6.10233	-6.04055	-5.9181	-18.7
356	-4.88389	-4.85222	-4.78924	-15.18
357	-3.66396	-3.65059	-3.62393	-11.6
358	-2.44292	-2.43896	-2.43104	-7.98
359	-1.22113	-1.22064	-1.21965	-4.333
360	0.001029	0.001029	0.001029	-0.67

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The graph of table 1 is showed below:

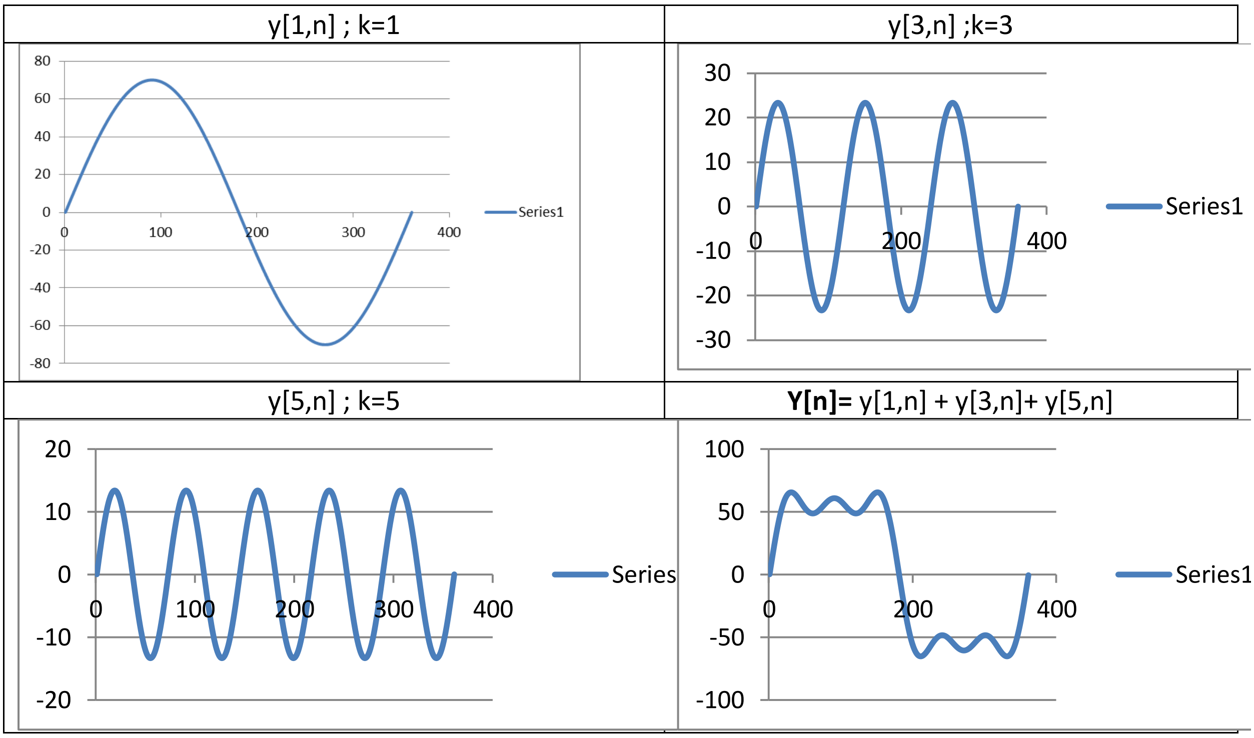

Figure 3. Graph of Odd Data Index

The example calculation for even value of frequency index (k = 2, 4, 6, 8, 10)

- Parameter forming sine signal, fo = 1Hz, A = 220, 2 = 1, n = 1..360, ts = 1/360 sec; so the equation y[2,n]

$$y[2,n] = \frac{220}{3.14*2} * \sin\left(2 * 3.14 * 2 * 1 * n * \frac{1}{360}\right), by\ \ n = 1..360$$

- Parameter forming sine signal, fo = 1Hz, A = 220, k = 4, n = 1..360, ts = 1/360 sec; so the equation y[4,n]

$$y[3,n] = \frac{220}{3.14 * 4} * \sin(2 * 3.14 * 4 * 1 * n * 1/360), by\ \ n = 1..360$$

- Parameter forming sine signal, fo = 1Hz, A = 220, k = 5, n = 1..360, ts = 1/360 sec; so the equation y[6,n]

$$y[5,n] = \frac{220}{3.14 * 6} * \sin(2 * 3.14 * 6 * 1 * n * 1/360), by\ \ n = 1..360$$

- Etcetera

the integrating signal Y[n] is summing from every y[k,n] value, here is the example data :

Table 2. Example Even Data Index

n	y[2,n] k=2	y[4,n] k=4	y[6,n] k=6	y[8,n] k=8	y[10,n] k=10	Y[n]= y[2,n]+ y[4,n]+ y[6,n]+ y[8,n]+ y[10,n]
0	0	0	0	0	0	0
1	1.222	1.2212	1.22	1.2183	1.216	6.097488
2	2.4425	2.4365	2.4266	2.4128	2.3952	12.11359
3	3.66	3.6399	3.6067	3.5605	3.5016	17.96869
4	4.873	4.8257	4.7473	4.6389	4.5017	23.58662
5	6.0802	5.9879	5.836	5.6271	5.3652	28.89634

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6	7.2799	7.121	6.8607	6.5059	6.0658	33.83336
7	8.4708	8.2194	7.8104	7.2583	6.5823	38.34116
8	9.6513	9.2778	8.6746	7.8694	6.8991	42.37222
9	10.82	10.291	9.4438	8.3276	7.0064	45.88901
10	11.976	11.254	10.11	8.6238	6.901	48.86459

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351	-11.03	-10.47	-9.573	-8.394	-7.003	-46.4734
352	-9.866	-9.466	-8.822	-7.965	-6.934	-43.0537
353	-8.687	-8.416	-7.975	-7.381	-6.655	-39.114
354	-7.498	-7.324	-7.04	-6.653	-6.174	-34.69
355	-6.3	-6.197	-6.028	-5.796	-5.506	-29.8274
356	-5.094	-5.04	-4.95	-4.827	-4.67	-24.5812
357	-3.882	-3.858	-3.818	-3.763	-3.693	-19.0144
358	-2.665	-2.657	-2.644	-2.627	-2.604	-13.197
359	-1.445	-1.444	-1.442	-1.439	-1.435	-7.20446
360	-0.223	-0.223	-0.223	-0.223	-0.223	-1.11579

The graph of table 2 is showed below:

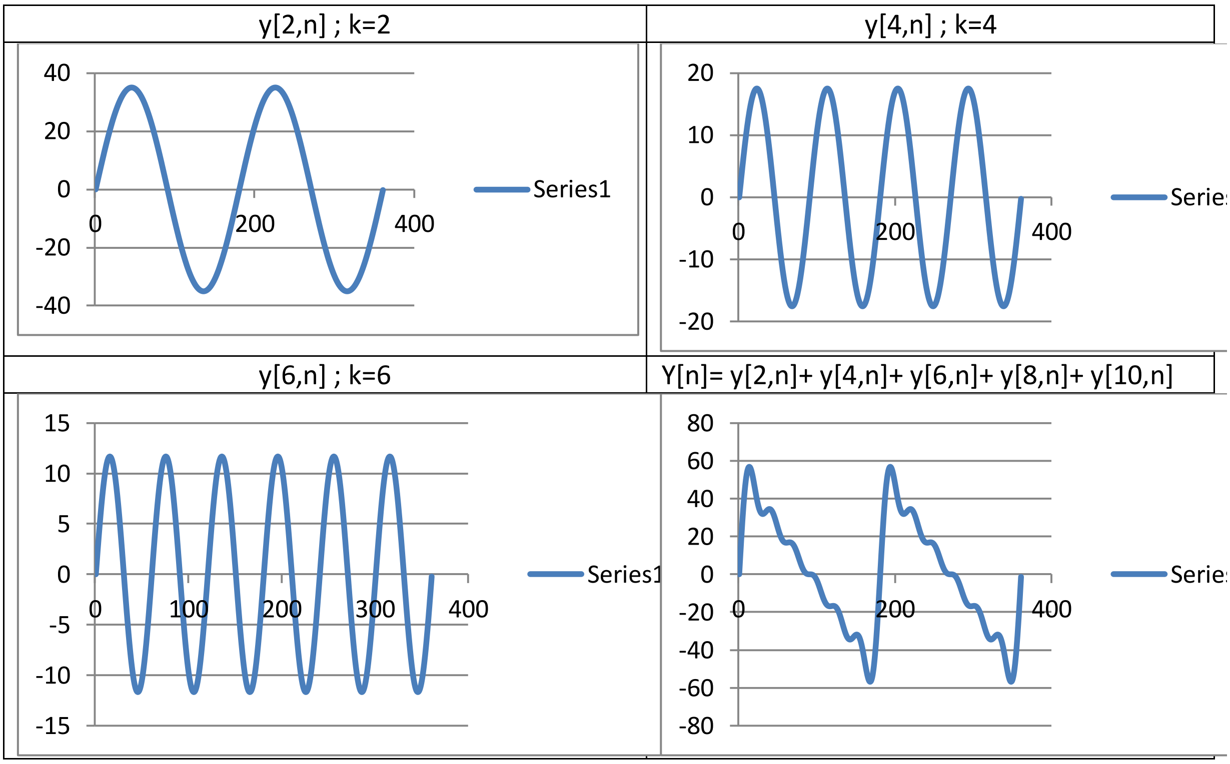

Figure 4. Graph off Even Data Index

For random data is not showed in this paper, due to space constraints, for convolution operation insyaAllah will be written on opportunity to come.

DECIPHERING SIGNAL

Decciphering of the integrated signal can be done with using the help of Fourier theory, in this case the most appropriate is a DFT (Discrete Fourier Transform). With discrete Fourier

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transformation we will obtain the frequency spectrum, amplitude spectrum, and the energy density spectrum is owned by the digital signal. The integrated wave signal data is obtained from integrating signals with using MS Excel application as described in the previous session. The next step dechipers the integrated signal data with using DFT equation 4, here is the DFT formulation:

$$Y(k) = \sum_{n=0}^{N-1} X(n).W_N^{kn}, \text{ for } W_N = e^{-j2\pi/N} \quad (4)$$

by :

Y(k) : frequency spectrum for frequency index (k)
X(n) : digital data
k : frequency index
n : sampling 0,1,2,3……,N-1
N : the number of sampling

Equation 4 in matrix can be writen :

$$\begin{vmatrix} Y(1) \\ Y(2) \\ Y(3) \\ . \\ . \\ . \\ Y(K) \end{vmatrix} = \begin{vmatrix} X(0) \\ X(1) \\ X(2) \\ . \\ . \\ . \\ X(N-1) \end{vmatrix} \begin{vmatrix} W_N^{0,1} & W_N^{0,2} & W_N^{0,3} & . & . & W_N^{0,K} \\ W_N^{1,1} & W_N^{1,2} & W_N^{1,3} & . & . & W_N^{1,K} \\ W_N^{2,1} & W_N^{2,2} & W_N^{2,3} & . & . & W_N^{2,K} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_N^{N-1,1} & W_N^{N-1,2} & W_N^{N-1,3} & . & . & W_N^{N-1,K} \end{vmatrix} \quad (5)$$

The imajener value of j==√-1 can not be processed in program coding, so we use Euler equation, following:

$$e^{-\frac{j2\pi kn}{N}} = \cos\left(\frac{2\pi kn}{N}\right) - j\sin\left(\frac{2\pi kn}{N}\right) \quad (6)$$

According to the IEEE standard [23], k is the frequency index and A is the magnitude measured amplitude value, here is the foemulation:

$$A(k) = \frac{2*|Y(k)|}{N} \quad (7)$$

$f(k)=k.f_0$

$I_H(k) = A(k)$

All of the formulation are arranged in flowchart, and then used as a reference for making code program, by using data Y[n] table 1 and table 2 will be obtained graph of integrated signal, frequency spectrum, and amplitude spectrum, here is the example result:

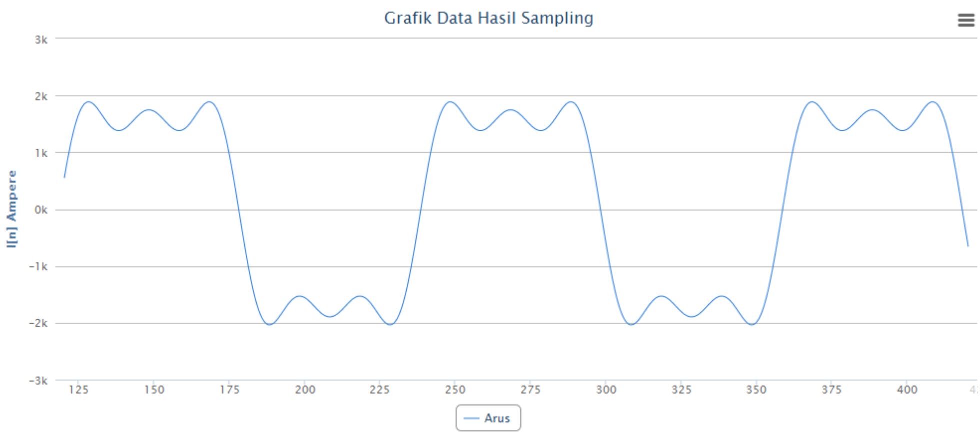

Figure 5 Graph from Y[n] Table 1

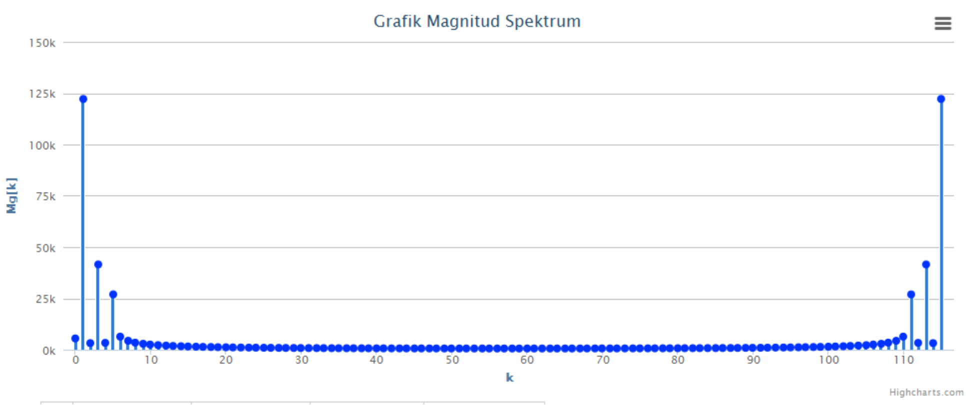

Figure 6. Frequency Spectrum of Y[n] Table 1

k	Real / Xr[k]	Imajiner / Xi[k]	Magnitudo / Mg[k]	Amplitudo /A
0	107.0714584	0	107.0714584	1.8460596275862
1	-559.85409724645	4033.1712861633	4071.8432231283	70.204193502211
2	95.376862296582	48.176521954468	106.85374644424	1.8423059731766
3	-560.92041892345	1265.9445580477	1384.6470093189	23.873224298602
4	53.811353469925	97.441117823051	111.31232278985	1.9191779791353
5	-585.19831123566	681.01782424992	897.90998458596	15.481206630792
6	-104.61295102739	184.73558539335	212.29956672891	3.660337357395

Figure 7. Amplitude Spectrum of Y[n] Table 1

Figure 6 and 7 show that are 3 spectrum values k=1, k=3, and k=5 have value more then zero, with using $f = kf_0$, we will obtain frequency:

$f_1 = 1*1 = 1Hz$
$f_2 = 3*1 = 3Hz$
$f_3 = 5*1 = 5Hz$

and the amplitude (A) on k=1 is 70.204, the amplitude (A) on k=3 is 23.873, the amplitude (A) on k=5 is 15.481.

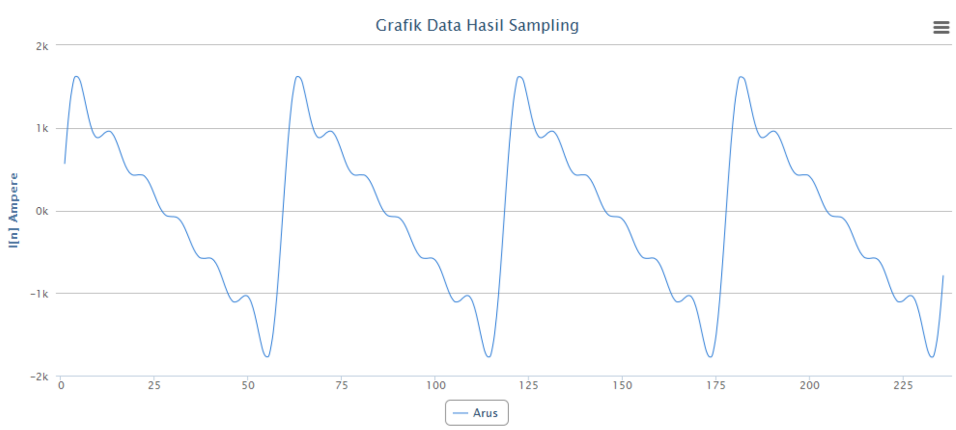

Figure 8. Graph from Y[n] Table 2

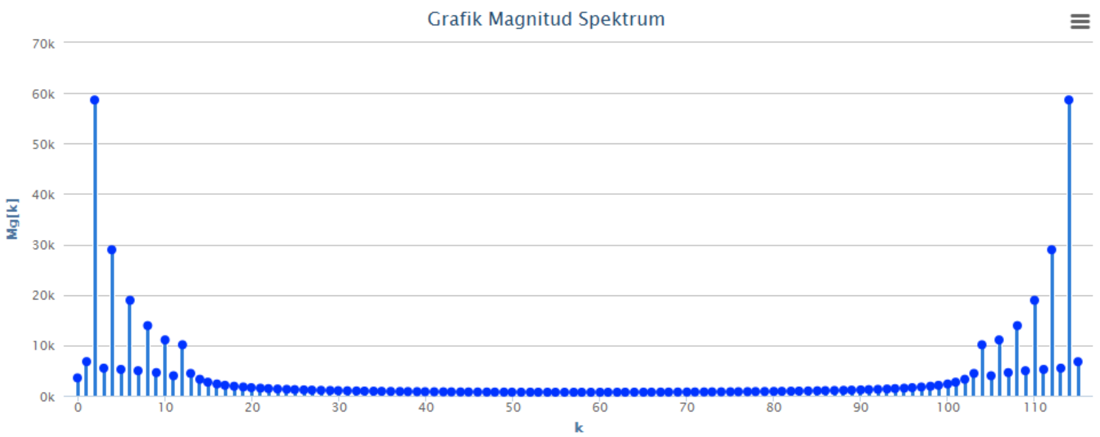

Figure 9. Frequency Spectrum from Y[n] Table 2

k	Real / Xr[k]	Imajiner / Xi[k]	Magnitudo / Mg[k]	Amplitudo /A
0	172.2674527	0	172.2674527	2.9701284948276
1	193.30716979306	-115.64564389607	225.25891069066	3.8837743222527
2	-466.39107060471	1897.8007504945	1954.2692545597	33.694297492409
3	181.22494939144	17.954254375744	182.11215646441	3.1398647666278
4	-449.34537311313	853.41279722123	964.48155337422	16.628992299555
5	157.61316924933	73.285401691218	173.81789672488	2.9968602883601
6	-422.56737986903	467.97177825979	630.52420713007	10.871107019484
7	123.35065281842	109.8625051692	165.18218303673	2.8479686730472
8	-388.77809365528	251.82500936851	463.21079591228	7.9863930329703
9	78.849130507863	130.80256759809	152.73014460838	2.633278355317
10	-353.3034958478	105.41456551657	368.69444096884	6.3568007063593

Figure 10. Amplitude Spctrum of Y[n] Table 2

The analysis and calculation of figure 9 and 10:

Figure 6 and 7 show that are 3 spectrum values k=2, k=4, k=6, k=8, and k=10 have value more then zero, with using $f = kf_0$, we will obtain frequency:

$f_2 = 2*1 = 1Hz$, $f_8 = 8*1 = 8Hz$

$f_4 = 4*1 = 3Hz$ $f_{10}=10*1=10Hz$

$f_6 = 6*1 = 5Hz$

and the amplitude (A) on k=2 is 33.694, the amplitude (A) on k=4 is 16.629, the amplitude (A) on k=6 is 10.87, the amplitude (A) on k=8 is 7.986, and , the amplitude (A) on k=10 is 6.357.

Total analysis of integrating and deciphering signal is writen on table 3, below:

Table 3. Analysis integrating and deciphering signal

Operator	The Integrated Signal (Y[n])			Deciphering Signal		Error (%)	
	Index (k)	Frequency (f) Hz	Amplitude (A)	Frequency (f)Hz	Amplitude (A)	Frequency	Amplitude
Summing	1	1	70.06369	1	70.204	0	0.200261
	3	3	23.35456	3	23.873	0	2.219866
	5	5	14.01274	5	15.481	0	10.47804
Summing	2	2	35.03185	2	33.694	0	3.818953
	4	4	17.51592	4	16.629	0	5.063508
	6	6	11.67728	6	10.87	0	6.913254
	8	8	8.757962	8	7.986	0	8.814402
	10	10	7.006369	10	6.357	0	9.268267
Summing	2	2	35.03185	2	34.01	0	2.916917
	4	4	17.51592	4	17.125	0	2.231798
	5	5	14.01274	5	13.982	0	0.219372
	7	7	10.0091	7	9.891	0	1.179926
	8	8	8.757962	8	8.564	0	2.214693
Multiplication	1	1	70.06369	1	71.223	0	1.654652
	3	3	23.35456	3	23.865	0	2.185612
	5	5	14.01274	5	14.453	0	3.141855
Multiplication	2	2	35.03185	2	35.412	0	1.085155
	4	4	17.51592	4	17.657	0	0.805439
	6	6	11.67728	6	11.881	0	1.744584
	8	8	8.757962	8	8.821	0	0.719779
	10	10	7.006369	10	7.103	0	1.379188
Multiplication	2	2	35.03185	2	34.571	0	1.315517
	4	4	17.51592	4	17.362	0	0.878743
	5	5	14.01274	5	13.926	0	0.619008
	7	7	10.0091	7	9.927	0	0.820254
	8	8	8.757962	8	8.539	0	2.500148

Total Error : 74.38919 %, Error average : 1.431%

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